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SOME NEW RESULTS ON GLOBAL NONEXISTENCE FOR ABSTRACT EVOLUTION WITH POSITIVE INITIAL ENERGY

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Dedicated to Olga Ladyzhenskaya with admiration and esteem

1. Introduction

In a recent paper [7] the problem of non-continuation was studied for abstract evolution equations of the type

$$Pu_{tt} + Q(t)u_t + A(t,u) = F(t,u), \quad t \in J = [0,\infty), \tag{1.1}$$

where P and $Q(t)$ are linear self-adjoint operators, and $A(t,u)$ and $F(t,u)$ are typically a divergence operator in u and a nonlinear driving force.

Other versions of (1.1) were considered earlier by Levine [3]–[6], for which he introduced the important technique of "concavity" analysis of auxiliary second order differential inequalities. In all these papers the principal mechanism of blow-up was the assumption of negative initial energy.

In an interesting paper [10], which has just appeared, Ono has also used concavity analysis to study blow-up, but in the more general case when the initial energy is allowed to take appropriately small positive values. His analysis primarily considers linear wave operators, and moreover is restricted to bounded

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