Topological **M**ethods in **N**onlinear **A**nalysis
Journal of the Juliusz Schauder Center
Volume 10, 1997, 195–223

ON LOCAL MOTION OF A COMPRESSIBLE BAROTROPIC VISCOUS FLUID WITH THE BOUNDARY SLIP CONDITION

Marek Burnat — Wojciech M. Zajączkowski

(*Submitted by A. Granas*)

Dedicated to O. A. Ladyzhenskaya

1. Introduction

We consider the motion of a compressible barotropic viscous fluid in a bounded domain $\Omega \subset \mathbb{R}^3$ with the boundary slip condition. Let $\rho = \rho(x,t)$ be the density of the fluid, $v = v(x,t)$ the velocity, $p = p(\rho(x,t))$ the pressure, $f = f(x,t)$ the external force field per unit mass. Then the motion is described by the following problem (see [3]):

$$(1.1)\qquad \begin{aligned} &\rho(v_t + v\cdot\nabla v) - \operatorname{div}\mathbf{T}(v,p) = \rho f && \text{in } \Omega^T = \Omega\times(0,T),\\ &\rho_t + \operatorname{div}(\rho v) = 0 && \text{in } \Omega^T,\\ &\rho|_{t=0} = \rho_0 \quad v|_{t=0} = v_0 && \text{in } \Omega,\\ &\overline{\tau}_\alpha\cdot\mathbf{T}(v,p)\cdot\overline{n} + \gamma v\cdot\overline{\tau}_\alpha = 0, \quad \alpha = 1,2, && \text{on } S^T = S\times(0,T),\\ &v\cdot\overline{n} = 0 && \text{on } S^T, \end{aligned}$$

where $\mathbf{T}(v,p)$ is the stress tensor of the form

$$(1.2)\qquad \begin{aligned} \mathbf{T}(v,p) &= \{T_{ij}(v,p)\}_{i,j=1,2,3}\\ &= \{\mu(\partial_{x_i}v_j + \partial_{x_j}v_i) + (\nu-\mu)\operatorname{div} v\,\delta_{ij} - p\delta_{ij}\}_{i,j=1,2,3}, \end{aligned}$$

1991 *Mathematics Subject Classification.* 35K50, 35F30, 76N10.

Key words and phrases. Compressible barotropic viscous fluid, local existence, potential theory, anisotropic Sobolev spaces with fractional derivatives.

The second named author was supported by Polish KBN Grant 2P03A 065 08.