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LERAY–SCHAUDER CONTINUATION THEOREMS IN THE ABSENCE OF A PRIORI BOUNDS

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Dedicated to O. A. Ladyzhenskaya

1. Introduction

In 1934, Leray and Schauder have published their fundamental paper *Topologie et équations fonctionnelles* [37], which is the founding father of algebraic topology in infinite-dimensional spaces and a milestone in nonlinear functional analysis and nonlinear differential equations. The style of this paper is still amazingly modern and its influence in contemporary mathematics considerable. This paper was among the thirty-seven most quoted mathematical papers for the period 1950–1965 and its influence still increased in the early seventies, with the development of bifurcation theory, global analysis and the use of topological techniques in critical point theory. The reader can consult the references [53, 32, 40] to get a first idea of the tremendous bibliography related to the consequences and extensions of [37], and the celebrated books of Ladyzhenskaya *et al.* for striking applications to Navier–Stokes equations [33] and to nonlinear elliptic [34] or parabolic partial differential equations [35].

The central topic of this paper is the study of *continuation theorems* for proving the existence of a solution to some equations. Let X and Y be topological spaces, $A \subset X$, and $f : X \to Y$, $g : X \to Y$ two continuous mappings. The fundamental idea of the *continuation method* to solve the equation

$$f(x) = g(x) \tag{1}$$

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