Topological **M**ethods in **N**onlinear **A**nalysis
Journal of the Juliusz Schauder Center
Volume 9, 1997, 29–39

GRADIENT ESTIMATES FOR SOLUTIONS OF THE NAVIER–STOKES EQUATIONS

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Dedicated to O. A. Ladyzhenskaya on the occasion of her seventy-fifth birthday

The motivation for this paper derives from the classical property of solutions of the Laplace equation

$$\Delta w = 0 \tag{1}$$

in a bounded domain Ω, that the gradient at interior points is bounded in magnitude, depending only on the distance d to $S = \partial\Omega$ and on a bound for $|w|$ on S. This property was extended by Odqvist [7], under some smoothness hypotheses on S, to solutions of the Stokes equations

$$\Delta\vec{w} = \nabla p, \tag{2}$$

$$\operatorname{div}\vec{w} = 0, \tag{3}$$

for slow stationary viscous fluid flows. We ask whether an analogous property also holds for solutions $\vec{w}(x)$ of the full Navier–Stokes equations for stationary viscous fluid flows

$$\Delta\vec{w} - R\vec{w}\cdot\nabla\vec{w} = \nabla p, \tag{4}$$

$$\operatorname{div}\vec{w} = 0. \tag{5}$$

1991 *Mathematics Subject Classification.* 35Q30, 76D05, 76D07.

The first author was supported in part by a grant from the National Aeronautics and Space Administration, and in part by a grant from the National Science Foundation. He wishes to thank Universität Leipzig for its hospitality during completion of the work.