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REVIEW

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In 1902, Bertrand Russell discovered a famous paradox of naive set theory—that for the set $S = \{x : x \notin x\}$, $S \in S$ if and only if $S \notin S$. The paradox comes from the fact that traditional set theory allows self-reference: once we have defined a new set (such as Russell's set S), we can then ask whether the new set is an element of itself. To avoid this (and similar) self-reference paradoxes and still allow mathematically useful constructions of sets of sets *etc.*, Russell introduced the following idea.

We start with known consistent sets of the standard type. Based on these sets, we can form new sets, *e.g.*, by considering all standard sets that satisfy a given property P: $\{x \text{ is of standard type} : P(x)\}$. By definition, these sets only contain sets of the standard type; so while we can legitimately consider a new set $S = \{x \text{ is of standard type} : x \notin x\}$, this set will contain only standard sets and not new sets (like the set S itself). These new sets form the first level in the set hierarchy.

We can also consider sets of such first level sets, *i.e.*, sets of the form $\{x \text{ is of first level} : P(x)\}$; these sets form the next (2nd) level, *etc.* Formally, we must explicitly assign to each variable x an integer value called its *type*, with the understanding that possible values of this variable x are sets of this type. The set $\{x^a : P(x^a)\}$ is then of type $a + 1$, the set $\{x^a : \forall y^b \exists z^c P(x^a, y^b, z^c)\}$ is of type $\max(a, b, c) + 1$, and the formula $x^a \in y^b$ only makes sense if $a < b$.

In this theory, if elements of the set A are of type a, then subsets of A are of type $a+1$. If elements of the set B are of type b, then pairs—*i.e.*, elements of the Cartesian product $A \times B$—have a type $\max(a, b)$. A function $f : A \to B$ is usually defined as a set of pairs $f \subseteq A \times B$,