The Review of Modern Logic
Volume 10 Numbers 3 & 4 (March 2005–May 2007) [Issue 32], pp. 91–111.

ARE THE NATURAL NUMBERS JUST ANY PROGRESSION? PEANO, RUSSELL, AND QUINE

JAN DEJNOŽKA

Are the natural numbers just any progression? It is widely held that Peano and Quine say yes, Russell no. For Russell criticizes Peano, and Peano and Quine criticize Russell.[1] The paper has four parts. (1) I describe Peano's theory as Russell understands it and as I think it is. (2) I describe Russell's criticism. (3) I extend Russell's criticism to odd counting procedures. (4) I discuss Quine's objections to Russell. I conclude that while it is not in the least controversial that infinitely many definitions of numbers and counting procedures are possible, Russell is right and my extension of Russell is right.[2]

1. Peano's Theory

Russell praises Peano's theory for being correct as far as it goes. He believes that it is an adequate theory of purely arithmetical equations such as $2 + 2 = 4$. He assigns it permanent value in the history of mathematics because it reduces arithmetic, and by extension all mathematics, to three undefined terms and five axioms.[3]

Received 12/27/2006.

[1][Russell 1903, 124–8], [Russell 1919, 5ff.], [Russell 1948, 236–7]. Peano returns the favor and criticizes Russell's definition; see [Russell 1903, 115]. On whether Peano invented the definition of number as a class of classes before Russell, see [Rodríguez-Consuegra 1991, 156–7] and [Kennedy 1974, 401–2].

[2]The seed of this paper was my e-mail discussion of Russell and Peano with Gregory Landini, Raymond Perkins, Torkel Franzén, Daniel Kervick, and Donald Stahl in Russell-l, the International Forum for Bertrand Russell Studies, `http://mailman.mcmaster.ca/mailman/listinfo/russell-l`, an "unadvertised" mailing list, from late 1999 to early 2000. I thank the group moderator for kindly granting me permission to make use of material from that discussion, which is in the Russell-l archives.

[3][Russell 1919, 5, 6–7]. Peano's original paper of 1889 uses four undefined terms and nine axioms [Kennedy 1980, 26]. Peano's axioms remain valuable for modeling subclasses of arithmetical truths not subject to Kurt Gödel's incompleteness theorem [Quine 1987, 16]; see [Kaye, 42–3]. Nearly all of the over 230 papers on Peano since 1940 are formal work on the axioms.