Volker Peckhaus, **Hilbertprogramm und kritische Philosophie**
(Göttingen, Vandenhoeck & Ruprecht, 1990).

A Review by C. Smoryński

The firm of Vandenhoeck and Ruprecht in Göttingen has recently published a number of fine books on the history of modern mathematics in Germany, with particular emphasis on Göttingen. Their works on David Hilbert are *Der Briefwechsel David Hilbert - Felix Klein (1886 - 1918)* (1985) edited by Günther Frei; *Über die Entstehung von David Hilbert's „Grundlagen der Geometrie"* (1986) by M.-M. Toepell; and now *Hilbertprogramm und kritische Philosophie* (1990) by Volker Peckhaus. The first of these has very little for the logician: a pair of letters (numbers 55 and 56) and an accompanying excerpt from a letter from Paul Gordan to Klein will satisfy those claiming a psychological background to Hilbert's programme. The book by Toepell should serve to dispel the myth that Hilbert's *Grundlagen der Geometrie* arose in a vacuum: Hilbert concerned himself with geometry throughout the 1890s, lecturing on it a number of times. However, except as background to the development of Hilbert's foundational views, Toepell's book is not essential reading for the historian of logic. This historian cannot, however, ignore the book of Peckhaus, which is the first booklength study of Hilbert's foundational activity in the period between Hilbert's *Grundlagen* and the end of the First World War. *A la* Toepell Peckhaus dispels the myth of Hilbert's 1904 - 1917 foundational vacuum; he offers the clearest perspective on the rôle of the paradoxes in Hilbert's programme I've ever seen; and he offers a very powerful challenge to those of us who stubbornly cling to the belief that there wasn't a philosophical bone in Hilbert's body.

All but the most perfunctory reader will notice a shift in Hilbert's attitude toward the paradoxes. When, in 1900, he originally called for consistency proofs for his axiomatisation of arithmetic and, eventually, for an axiomatisation of set theory, the paradoxes were not a major concern to him. Indeed, of his two publications on the matter, only the one— his famous address on problems— even hinted at the existence of the paradoxes when he mentioned that some collections— like that of all alephs— are inconsistent. Hilbert had already known of the set-theoretic paradoxes for several years. In his 1904 lecture at Heidelberg, he actually used the word "paradox" and declared that "the avoidance of such contradictions and the clarification of those paradoxes is rather to be kept in the eyes as a principal goal from the start in the investigation of the number-concept". Noting that logic depends on arithmetic and arithmetic on logic, he added that "for the avoidance of paradoxes a piecewise simultaneous development of the laws of logic and arithmetic is therefore necessary". It is not at all clear from these remarks whether the paradoxes are now an impetus or merely a constraint. In his later popular accounts of his foundational views, the paradoxes would become not merely an impetus, but the prime mover. Indeed, in his 1917 lecture on axiomatic thinking Hilbert was so taken with the paradoxes that he considered them to be the reason Kronecker and Poincaré rejected Cantor's set theory. Dropping the chronologically absurd reference to Kronecker, Hilbert would again wax poetic over the paradoxes in his other great popular account— his 1925