J. Chapman and F. Rowbottom *Relative Category Theory and Geometric Morphisms: A Logical Approach* Oxford Logic Guides 16, Oxford, Clarendon Press, 1992

Reviewed by

Colin McLarty
Department of Philosophy, Case Western Reserve University,
Cleveland, OH USA 44106

This book aims to make topos logic an adequate tool for all topos theory by extending it to handle category theory in toposes, or "relative category theory", with the relative Giraud theorem as test case. The problem and the test were remarked as early as Johnstone [*1977*, xviii].: "the formal language approach breaks down when confronted with the relative Giraud theorem; whilst [it] is a very powerful tool in proofs within a single topos, it is not well adapted to proofs in which we have to pass back and forth between two toposes by a geometric morphism."

It is well known that each topos S has an *internal languaqe* called L_S, a multi-sorted constructive set theory interpreted in S. For any topos S, Heyting's intuitionistic predicate logic is sound in L_S, but classical logic generally is not. Of course classical logic ls sound in some toposes, such as the topos of classical sets, **Set**. But in general the law of excluded middle and the axiom of choice fail.

There are different views on the internal language. Most category theorists are not logicians and many dislike the syntactic details needed to make it a rigorous tool. Barr and Wells [*1985*] avoid it almost entirely. On the other hand Bell [*1988*] introduces toposes almost entirely in terms of it. Chapman and Rowbottom justly say their book "is essentially self-contained, except for basic category theory, which may be found in Mac Lane [*1971*] or Barr and Wells [*1985*], Chapter 1. However, it forms a natural sequel to Bell's book [*1988*]" (p.7).

Their task falls into two parts: treating small categories in the internal language, and treating certain large ones. A *small category* in a topos is with an object of objects and an object of arrows. In the topos **Set** then, it is a category with a set of objects and a set of arrows. A large category is one too big to be small. In **Set** it is a category with a proper class of objects and of arrows. The theory of small categories in **Set** has always been largely constructive and so works in any topos. But expressing it in the interval language has been surprisingly thorny.

The composite of arrows f and g is defined if the codomain of f is the domain of g. Logicians usually formalize partial functions by relations. Instead of a term gf for "the composite of f and g" we use a relation $C(f, g; k)$ read "k is the composite of f and g". The definability condition for composites is then stated

$$(\exists k)\, C(f, g; k) \leftrightarrow \mathrm{Dom}(g) = \mathrm{Cod}(f)$$

with the functions Dom and Cod for domain and codomain. But simple formulas become