## X Modern Logic ω

Richard L. Epstein and Walter A. Carnielli, *Computability: Computable Functions, Logic, and the Foundations of Mathematics*, Belmont, California, Wadsworth, Inc., 1989. xvii + 297 pp.

## Reviewed by

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What makes recursion theory tick? Why did the field develop — what are the themes that underlie the subject? Three major threads running through recursion theory come to mind quickly:

- (A) Considerations of machine computability, in evidence from Turing's 1936 paper that introduced Turing machines, right up to current work on computational complexity.
- (B) Logical/foundational matters, well exemplified by Gödel's use of the class of primitive recursive functions in his 1931 paper on undecidability.
- (C) "Mainstream" pure mathematics. For example, the opening sentence of the first book ever written on the subject, Péter's Recursive Functions, reads: "The theory of recursive functions properly belongs to number theory; indeed, the theory of recursive functions is, so to speak, the function theory of number theory."

Corresponding to these three threads are the groups of people most interested in recursion theory, respectively, computer scientists, logicians (both mathematical and philosophical), and mathematicians. Of course, the threads are often closely interwoven. The very title of Turing's abovementioned article, "On Computable Numbers, with an Application to the Entscheidungsproblem", suggests the interplay among (A), (B), and (C) present in that paper.

The textbook by Epstein and Carnielli has a much shorter title, Computability, which hints at an emphasis on thread (A). But the subtitle tells the true, thread (B) story. This book is indeed about Computable Functions, Logic, and the Foundations of Mathematics and their mutual connections. In pursuing that theme, the authors develop the material in a strikingly original manner. A recursion theory textbook which waits until page 63 (out of