REFLECTIONS ON THE INTERPLAY BETWEEN MATHEMATICS AND LOGIC

Department of Mathematics and Statistics
McMaster University
Hamilton, Ontario L8S 4K1, Canada

1. Introduction.

Van Heijenoort has rightly stressed that, in the work of Frege and Russell, logic is universal; neither logician made use of partial universes of discourse, as did Boole and De Morgan, but only *the* universe of everything. One central consequence of this universality, he has stressed as well, is that neither Frege nor Russell raised any metalogical or metamathematical question – nothing about the consistency or independence of the axioms of logic, or about their completeness (van Heijenoort *1967a*, 326).

Let us give an example, one not mentioned by van Heijenoort, of Russell and Whitehead's assertion that it is not possible to stand outside logic. The authors of *Principia Mathematica* insisted that the Principle of Mathematical Induction cannot be used to prove theorems *about* their system of logic (Whitehead and Russell *1910*, 135). This is in utter contrast with work in logic for the last fifty years. A great variety of different kinds of induction (e.g. on the length of formulas, on the kinds of formulas, etc.) are now standard textbook devices for proving theorems about first-order logic, particularly since terms and formulas are given by recursive definitions.

Van Heijenoort contrasts the approach of Frege and Russell with that of Löwenheim (*1915*), who belongs to the Peirce-Schröder tradition, and notes that the latter tradition used universes of discourse. It is precisely the metalogical questions that Löwenheim considers (van Heijenoort *1967a*, 327; *1977*, 183). But let us be clear that the metalogical notions which Löwenheim uses are those of validity and satisfiability, not those of con-