Ernst Schröder and the Distribution of Quantifiers

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The third volume of Schröder's Vorlesungen über die Algebra der Logik (Exakte Logik) is entitled Algebra und Logik der Relative; it presents the logic of relations in the framework of the algebra of logic. The only extant part of this volume is the "Erste Abteilung", being the last part of the Vorlesungen published by Schröder himself: with 1895 as the year of its appearance, it precedes the second "Abteilung" of the second volume edited by Eugen Müller in 1905 (after Schröder's death in 1902), and no further "Abteilung" followed.

Among the twelve lectures which form the first "Abteilung", the second lecture provides "the formal foundations, in particular of the algebra of binary relatives". In § 3, Schröder introduces the "fundamental stipulations", including rules for the use of the signs \sum and \prod for product and sum ("which were omitted at the beginning", as Schröder says on p. 35). These signs correspond to our modern universal quantifier (\bigwedge or \forall) and existential quantifier (\bigvee or \exists), and propositional schemata containing them belong to "quantificational logic". But Schröder does not delimit such a separate domain; he regards $\prod_x Ax$ and $\sum_y By$ as generalizations of $A_1A_2 \ldots A_m$ and $B_1 + B_2 + \ldots + B_n$, respectively, and treats propositional schemata containing them within his "propositional calculus" ("Aussagenkalkül") without much ado.

¹Schröder, Ernst: Algebra und Logik der Relative, der Vorlesungen über die Algebra der Logik dritter Band. Erste Abteilung. B.G. Teubner: Leipzig 1895. The réprint (as "second edition" by Chelsea Publishing Company (Bronx, N.Y. 1966) contains some changes (see below, note 7) and is conjoined with Ernst Schröder, Abriß der Algebra der Logik. Bearbeitet im Auftrag der Deutschen Mathematiker-Vereinigung von Eugen Müller [Teil I und II], Leipzig 1909/1910. On Müller's editorial work on the logical parts of Schröder's scientific estate see Volker Peckhaus, "Karl Eugen Müller (1865–1932) und seine Rolle in der Entwicklung der Algebra der Logik", History and Philosophy of Logic 8 (1987), 43–56.