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REVIEW

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The Classical Decision Problem first appeared in a 1997 hard-cover edition within the Springer series *Perspectives in Mathematical Logic.* The book under review is a soft-cover reissue within Springer's *Universitext* series.

The decision problem for first-order predicate logic—Hilbert's *Entscheidungsproblem*—is this: Does there exist an effective procedure for deciding whether an arbitrary first-order sentence S is logically valid or, alternatively, whether S is satisfiable. These alternative formulations are equivalent given that S is valid if and only if $\neg S$ is unsatisfiable. Assuming that effectiveness is captured by the technical notion of partial recursive function (the Church–Turing Thesis), it was shown in the mid-1930s by Church and also by Turing that there is no effective decision procedure of the desired sort (Church–Turing Theorem). In light of this negative result, one proceeds to ask about subclasses C of the collection of all first-order sentences, although, in general, a procedure for deciding validity for sentences in C might exist in the absence of a procedure for satisfiability for C. Similarly, demonstrating that there is no decision procedure for validity for C is compatible with the existence of a decision procedure for satisfiability for C. In any case, the *Entscheidungsproblem* is now recast as a classification problem: Which subclasses are decidable for satisfiability (validity) and which are undecidable?

This classification problem is now completed—at least if one considers standard subclasses only, *i.e.*, those determined by quantifier prefix and by which predicate and function symbols may occur. After an introductory first chapter, Part I of the book, comprising Chapters 2–5, is concerned with (minimal) undecidable subclasses, where "undecidable" tends to mean "no decision procedure for satisfiability." Part II