

John N. Crossley, Jeffrey B. Remmel, Richard A. Shore, Moss E. Sweedler (eds.), *Logical Methods: In Honor of Anil Nerode's Sixtieth Birthday*, Boston, MA, Birkhäuser Boston, 1993. x + 813 pp.

Reviewed by

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Anil Nerode was born in 1932, at about the same time as recursion theory itself came to life as an independent field. In 1992 a conference was held at Cornell University to celebrate Nerode's 60th birthday. *Logical Methods* is the proceedings of that conference.

Over the years Nerode's research has reached in various directions; often, though by no means always, his work has ultimately had its roots in the area of recursive equivalence types (RET's) and isols. J. C. E. Dekker initiated the study of RET's in 1953. Two sets of natural numbers A, B are said to be *recursively equivalent* if $f(A) = B$ for some 1-1 partial recursive function f whose domain contains A . Thus *recursive equivalence types*, the equivalence classes of this relation, form a recursion-theoretic version of cardinal numbers. As with the cardinal numbers, one may define addition, multiplication, an order relation, and so on. The algebraic and order structures of the RET's are much more complex than the Cantorian originals, however.

Of particular interest are the RET's that behave like finite cardinals. If a set A is not recursively equivalent to any of its proper subsets, A is called *isolated* and its RET is called an *isol*. The isolated sets are easily seen to be precisely those sets with no infinite recursively enumerable subsets. Like the whole set of RET's, the isols form a very rich system as far as algebraic and order properties. For example, there exists an uncountable collection of mutually incomparable isols. However, isols are much more tractable than RET's in general. In particular, they may be shown to share many properties with the natural numbers.

How does one show that? In the latter half of the 1950's, John Myhill got the ball rolling in this direction with his work on combinatorial functions and combinatorial operators. Every function f from N to N