

NOTES AND QUERIES

WHO INVENTED CANTOR'S BACK-AND-FORTH ARGUMENT?

Charles L. SILVER

Department of Computer Science 527
Southeastern Louisiana University
Hammond, LA 70402, USA

One reads that Peano's postulates are really Dedekind's and Dedekind's chains are really Frege's and Newton's method was known to Archimedes. As Paul J. Campbell has noted [1978, 85], "the history of mathematics is rife with a variety of misattributions, whose continued propagation by oral and written tradition is variously due to widespread ignorance of the historical facts, accepted convention, or just plain complication of the situation." This paper raises the question: Who Invented Cantor's Back-And-Forth Argument?

Cantor's Back-And-Forth Argument (BAF) has emerged as a vital technique in the theory of models. Not to be confused with his Diagonal Argument, which is used most frequently to prove that there are more real numbers than rationals, Cantor's BAF argument is best known for its use in proving the isomorphism of any two countable, dense linear orders (without endpoints). It is a well-entrenched belief among set-theorists and model-theorists that BAF is due to Cantor, who first introduced it to prove that result. Indeed, Cantor is sometimes called "the father of the back and forth argument" (see, e.g., [Barwise 1973, 5-6]), and is frequently cited for proving the isomorphism theorem using BAF (see, e.g., [Dickmann 1985, 348]; [Roitman 1990, 123]; [Shapiro 1991, 160]; and van Dalen [1983, 132]). Such citations, as in the case of most folkloric attributions, however, fail to provide precise bibliographic references.

Many years ago, while in graduate school, I discovered a proof of the isomorphism theorem that did not require the BAF technique. Since this simplification was surprising to several logicians, I pursued the matter a bit at that time. That led me to look up Cantor's famous proof. I was shocked to discover that Cantor did the proof "my way," not using the BAF technique which has become associated with him. So, at that time, two facts were surprising: (1) the isomorphism result did *not require* BAF; and (2) Cantor did not use BAF himself, at least not where he supposedly introduced it (namely, in the two-part