ℵ MODERN LOGIC ω

HERBRAND'S FUNDAMENTAL THEOREM AND THE BEGINNING OF LOGIC PROGRAMMING*

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INTRODUCTION.

In this paper we argue that in his 1930 thesis, Jacques Herbrand developed the concept of unification central to automatic theorem proving and logic programming, not peripherally as has been suggested, but as an element essential to the proof of his Fundamental Theorem, hereafter abbreviated, FT.

Excellent surveys of Herbrand's work can be found by Goldfarb in [Herbrand *1971*, 1–20], in Jean van Heijenoort [*1967*, 525–529; *1968*; *1986*], and Irving Anellis [*1991*; *1992*]. Herbrand set out to develop a unified approach to proof theory. His method of investigation, involving the notions of completeness, consistency, and decidability, was directed toward answering the question: what finite sense can generally be ascribed to the truth property of a formula with quantifiers, particularly the existential quantifier, in an infinite universe? The existential quantifier, interpreted as standing for a choice function, posed the main difficulty. Since it is not generally replaceable by a computable function, it is not always possible to constructively instantiate these quantified variables.

The major influence on Herbrand's development of unification came from Russell and Whitehead's *Principia Mathematica*, hereafter abbreviated, PM. Herbrand, like Hilbert, used PM as the example that classical mathematics can be codified and presented as a formal system. His key concept of a normal identity and what he refers to as Property *A*, are derived from a method used by Russell and Whitehead to construct quantificational logic. This method appears in Herbrand's 1928 paper, "On Proof Theory" [Herbrand, 29–32]. As Goldfarb [Herbrand, 4] writes,

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