

Michael Dummett

Elements of Intuitionism. Second Edition.

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REVIEW

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The first edition of this book appeared as the second volume in the series, Oxford Logic Guides, in 1977 and became the standard introduction to the intuitionistic philosophy of mathematics very soon. The present edition satisfies the highest standards of publishing quality, compared to the rather poor features of the former printing. The second edition preserves the original structure of the content adopted by the author for the first edition. Chapter 1 outlines preliminaries concerning constructive proofs, the meaning of logical constants, a sample of logical laws typical for intuitionism, and functional completeness. Elementary intuitionistic mathematics (arithmetic, real numbers, order, the axiom of choice) is discussed in Chapter 2. More advanced topics (infinity, the fan theorem, bar induction, the continuity principle, the Bar Theorem and its proof by Brouwer, continuous functionals and their representation, the uniform continuity theorem) are presented in Chapter 3. The next chapter is devoted to logical matters (natural deduction, the sequent calculus, cut-elimination, decidability of intuitionistic sentential logic, normalization). Chapter 5 contains an exposition of semantics for intuitionistic logic (valuation systems, lattices and finite model property, topological spaces, Beth trees, the semantics for intuitionistic predicate logic, the completeness of intuitionistic predicate logic, generalized Beth tree, compactness). Intuitionistic formal systems, realizability, and the Creative Subject (an idealized mathematician used in thought-experiments for motivating axioms and basic principles of mathematical reasoning) are discussed in Chapter 6 (under the title “Some further topics”). Concluding philosophical remarks (philosophical foundations of constructive mathematics, the notion of proof, partial functions, logical constants as represented on Beth trees,