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Review of

CRAIG SMORYŃSKI, *LOGICAL NUMBER THEORY I, AN INTRODUCTION*

Berlin-Heidelberg-New York: Springer-Verlag Universitext, 1991
x + 405 pp. ISBN 3-540-53346-0 and ISBN 0-387-52236-0

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Smoryński's account of what he calls 'logical number theory' is an entertaining account of some of the germs in mathematical logic and 'arithmetic', *i.e.*, the theory of the natural number $\mathbb{N}$, up to around 1970. The mathematics is sometimes very beautiful, the presentation of it is always personal and highly idiosyncratic, and there are a great number of remarks putting the material in its historical context. It is intended to be read by anyone with sufficient mathematical background, such as an advanced undergraduate, or a beginning postgraduate. Such a reader will find a combination of material from logic and number theory with some interesting historical disgressions.

The title suggests that the book is concerned with number theory first and foremost, and in particular those aspects of it that are best studied using tools from a logician's toolbox. The author has therefore been able to make a rather free (and often refreshingly different) choice of material. Typical here is the discussion of the Fueter-Pólya theorem very early on in the book. This theorem concerns polynomials $P(x, y)$ that define bijections from $\mathbb{N}^2 \to \mathbb{N}$. One such polynomial, called the *pairing function* was defined by Cantor, and is essentially $\langle x, y\rangle = (x^2+2xy+y^2+3x+y)/2$. (Here, unlike, Cantor, we take the convention that 0 is a natural number, so $\mathbb{N} = \{0, 1, 2, 3, \ldots\}$, and the definition of Cantor's pairing function has been modified accordingly.) The Fueter-Pólya theorem states that the only quadratic polynomials $P(x, y)$ over the reals defining a bijection $\mathbb{N}^2 \to \mathbb{N}$ are the Cantor pairings $\langle x, y\rangle$ and $\langle y, x\rangle$.

Of course, the pairing function or something like it is essential for any study of logical number theory, but the Fueter-Pólya theorem, however interesting it is, is not at all necessary, and is not referred to later on in the book. Nevertheless, the issues it raises are fascinating and