

Review of
**JON BARWISE AND LAWRENCE MOSS, *VICIOUS
CIRCLES. ON THE MATHEMATICS OF
NON-WELLFOUNDED PHENOMENA.***

Stanford: CSLI Publications, 1996

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In 1988, Peter Aczel published his now famous *Non-Well-Founded Sets*, where he introduced his Anti-Foundation Axiom AFA. Twelve years later, Jon Barwise and Lawrence Moss publish, in the same series, their *Vicious Circles*, which is also devoted to anti-foundation and non-well-founded sets.

This very pleasant book has been written for a wider readership than *Non-Well-Founded Sets*: students or researchers in logic, computer science and philosophy. Besides the technicalities of anti-foundation, it describes a wide range of applications of non-well-founded sets. Its style is clear and lively; mathematics are introduced and motivated by means of well chosen examples. The book can be used as a classroom textbook or for individual study; it contains many (solved) exercises. It is of a particular interest for researchers who want to start working on anti-foundation: it presents recent results, sometimes not yet published, and introduces some research projects.

Vicious Circles is divided into six parts. Parts I and II introduce the background set theory and circularity in various domains. Then parts III and IV present the basic theory and elementary applications. Finally, advanced theory and applications are described in parts V and VI.

Let us review each part in turn. Part I is a very short introduction to set theory (the reader is expected to have read an introduction to ZFC elsewhere). It gives the axioms of the underlying set theory: ZFC⁻, that is ZFC with atoms (*Urelemente*), but without foundation, of course. This explicit use of atoms is much clearer than in Aczel's book.