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REVIEW

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What is “automated reasoning”? Crudely speaking, *automated reasoning* is an area whose main objective is to make computers solve hard open problems in mathematics and in other well-defined formal areas.

Is it successful? Oh yes, there have been quite a few success stories when automated reasoning programs succeeded in solving long-standing open mathematical problems.

Probably the most well-known of these problems is the antiautomorphism semigroup problem. This problem was originally formulated by a well-known algebraist I. Kaplansky as a challenge to the automated reasoning community. Let S be a *semigroup*, *i.e.*, a set with an associative operation $*$. A mapping $f : S \rightarrow S$ is called an *antiautomorphism* if $f(x * y) = f(y) * f(x)$ for all x and y . Many semigroups have antiautomorphisms: *e.g.*, for every natural number n , transposition is an antiautomorphism on the semigroup of all $n \times n$ matrices. This particular antiautomorphism is an *involution*, *i.e.*, $f(f(x)) = x$ for every x . At the time when Kaplansky formulated this question, in every known finite semigroup with an antiautomorphism there was also an antiautomorphism which is an involution. It was therefore conjectured that every finite semigroup with an antiautomorphism has an antiautomorphism which is an involution. Mathematicians tried hard, but could neither prove this conjecture nor find a counterexample.

An automated reasoning program succeeded in finding a non-trivial counterexample — a semigroup of 83-th order!