The Review of Modern Logic
Volume 9 Numbers 1 & 2 (November 2001–November 2003) [Issue 29], pp. 95–113.

VIEWS ON THE REAL NUMBERS AND THE CONTINUUM

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1. Introduction

The history of the real numbers is related to many areas of mathematics, sometimes in a central way and sometimes in a tangential way. Such a pervasive topic is itself quite complex and can be thought of as three problems:

- What is a real number?
- Are the real numbers and the real line one and the same?
- What is a continuum?

In the first approaches to the real numbers, mathematicians used these three concepts — real number, real line, and continuum — interchangeably, as the early view of the real numbers was geometric. Mathematicians spoke of the "law of continuity" or a continuous variable or magnitude with the image of the real line in mind. They cast arguments in calculus in this geometric language. However, this dependence on geometry was unsatisfactory to some mathematicians by the early 19th century. To provide logical rigor to the proofs in calculus, some realized the need for an algebraic or arithmetic description of the real numbers. In this paper, we examine how three theories define in algebraic form a characteristic of the real number system which distinguishes it from the rational numbers, how the authors of these theories dealt with the issue of the identification of the real numbers with the real line, and finally how the authors understood the continuum.

2. Defining Characteristics of the Real Numbers

In the second half of the nineteenth century, three main theories of the real numbers were developed. These are the theories of Richard Dedekind (1831-1916), Georg Cantor (1845-1918) and Eduard Heine (1821-1881), and Karl Weierstrass (1815-1897). In each case, the authors of these theories assumed that the rational numbers were well-understood and used the rational numbers as the starting point for