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META INDUCTION IN OPERATIONAL SET THEORY

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§1.Introduction. Set theory is usually organized as a first order theory where the variables range over a collection or universe of sets. This universe is assumed to be a well defined totality, for the theory involves global quantifiers $(\forall Y)$ and $(\exists Y)$ that range over the universe. The global quantifiers *induce local quantifiers* $(\forall Y \in X)$ and $(\exists Y \in X)$.

On the other hand, operational set theory (see [6]) rejects the universe of sets as a well defined totality and in principle questions the legitimacy of the global quantifiers. Noting that a first order theory without global quantifiers is difficult to handle, we have tried to avoid a complete rupture by allowing global quantifiers under some restrictions. For example, in [6] we restrict the global quantifiers by imposing a general control of such quantifiers under intuitionistic logic. We do not support anymore this type of restriction, as we prefer to preserve classical logic for the whole system, and we have chosen to impose restrictions on the axiomatic structure of the theory.

The situation of the local quantifiers (that in standard set theory are induced by the global quantifiers) is different. In fact, we assume, and we require, that each set in the universe is a complete totality that supports local quantification. As we explain below, there is a price to pay for this requirement, concerning how sets in general are allowed to enter the theory.

1.1. We are now in position to give a rough description of what we understand by operational set theory. It is a first order theory involving set operations, set predicates, classical connectives, local quantifiers and global quantifiers. Furthermore, the axioms of the theory are *operational*, and this means that each axiom is a closed formula of the form $(\forall Y_1)...(\forall Y_n)\phi$ is where ϕ is a local formula (no global quantifiers) and $0 \leq n$. Usually, we identify the axioms via the local formula ϕ.

1.1.1. We require that the sets in the theory be introduced with (operational) axioms, but we also require that whenever a set is introduced some argument or construction is provided that shows that the set is a well defined totality that supports local quantification. For