Modern Logic
Volume 8, Number 3 & 4 (May 2000–October 2001), pp. 7–19.

THE ENIGMA OF THE INFINITESIMAL: TOWARD CHARLES L. DODGSON'S THEORY OF INFINITESIMALS [1]

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1. Introduction

In 1888 Charles L. Dodgson published *Curiosa Mathematica, Part I. A New Theory of Parallels* (hereafter CMI) containing his novel alternative Euclidean parallel postulate: in every circle the inscribed equilateral hexagon is greater [in area] than any one of the segments which lie outside it. In Appendix II, "Is Euclid's Axiom True," he discusses infinitely large and infinitely small magnitudes, particularly infinitesimal lines and strips, and infinitesimal angles and sectors. In this section he develops a theory of infinitesimals that although flawed, contains elements that ultimately were addressed in the rigorous theory of infinitesimals Abraham Robinson created more than a half century later.

To provide a context for Dodgson's work, the paper begins with a survey of the main lines of thought about infinitesimals in the nineteenth century in analysis and in geometry, including the incomplete and divergent view held by Charles S. Peirce.

2. Infinitesimals in Analysis.

In the period between Gottfried Leibniz (1646–1716) and Abraham Robinson (1918-1974), roughly 300 years, infinitesimals were used in mathematics without being properly understood. In the first 200 years or so after the invention of the calculus infinitesimals as numbers were sometimes confused with the number zero. Alternatively, mathematicians following Augustin Cauchy (1789-1857) regarded them as variables with zero as their limit. The lack of a precise definition of the real number system was the principal stumbling block. Once this piece was put into place in the 1870s, the evolution of the calculus into a

[1] This paper is an expansion of the invited address, "Charles L. Dodgson's Theory of Infinitesimals" given at the 1998 Midwest Conference on the History of Mathematics held at Iowa State University.