Corrigenda: On the product theory of singular integrals

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We wish to acknowledge and correct an error¹ in a proof in our paper On the product theory of singular integrals, which appeared in Revista Matemática Iberoamericana, volume 20, number 2, 2004, pages 531-561. In Lemma 2.3.2, part (a), we wish to show that for $\lambda > 0$, the operator $R(\lambda, \mathcal{L}) = (\lambda I + \mathcal{L})^{-1}$ is bounded on $L^{\infty}(M)$ with a norm that may depend on λ . We write the operator as

$$R(\lambda, \mathcal{L})[f](x) = \int_M f(y) \, r_\lambda(x, y) \, dy.$$

It then follows that

$$r_{\lambda}(x,y) = \int_{0}^{\infty} e^{-\lambda s} H(s,x,y) \, ds$$

where H(s, x, y) is the heat kernel for the operator \mathcal{L} . We assert in equation (2.12) that there is a constant C so that for all $\lambda > 0$

(2.12)
$$|r_{\lambda}(x,y)| \le C \frac{d^2(x,y)}{V(x,y)}.$$

This is not correct when M is a compact manifold. It should be replaced by the statement that when M is compact and of dimension at least 3, there is a constant C so that for all $\lambda > 0$

(2.12a)
$$|r_{\lambda}(x,y)| \le C \frac{d^2(x,y)}{V(x,y)} + \frac{C}{\lambda}.$$

This estimate still shows that the operator $R(\lambda, \mathcal{L})$ is bounded on $L^{\infty}(M)$ if M is compact.

 $^{^1\}mathrm{We}$ wish to thank Professor Dachun Yang of Beijing Normal University for bringing this error to our attention.