Harnack's inequality for solutions of some degenerate elliptic equations

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Abstract

We prove a Harnack's inequality for non-negative solutions of some degenerate elliptic operators in divergence form with the lower order term coefficients satisfying a Kato type condition.

1. Introduction.

In this paper, we study the behavior of solutions of certain degenerate elliptic equations $Lu = 0$, where L is the operator

$$L := -\sum_{i,j=1}^{n} \frac{\partial}{\partial x_i}\Big(a_{ij}(x)\,\frac{\partial}{\partial x_j}\Big) + \sum_{i=1}^{n} b_i(x)\,\frac{\partial}{\partial x_i} + V(x)\,.$$

The coefficients a_{ij} are real-valued measurable functions whose coefficient matrix $A(x) := (a_{ij}(x))$ is symmetric and satisfies

$$(1.1) \qquad \omega(x)\,|\xi|^2 \leq \langle A(x)\,\xi,\xi\rangle \leq \upsilon(x)\,|\xi|^2\,.$$

Here $\langle\cdot,\cdot\rangle$ denotes the usual inner product on $\mathbb{R}^n$, and υ,ω are non-negative functions which will be described below.

Let us fix some notations that will be used throughout the paper. For functions f and g, we shall write $f \lesssim g$ to indicate that $f \leq Cg$ for some

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