Rev. Mat. Iberoamericana **18** (2002), 99–114

# A Regularity Theorem for Curvature Flows

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## 1. Introduction

The regularity theory of minimal surfaces and minimizers of other elliptic functional is well known, due to DeGiorgi [DG], Federer and Fleming [FF], Reifenberg [R] and F. Almgren [A]. However the only regularity theory of evolutionary problem is due to K. Brakke [B] about mean curvature flow of unit density surfaces. Here we prove a regularity theory for general surface flows with a similar density condition (Section 2, Definition 9). The evolutions that we deal with do not necessarily come from a gradient flow of some functionals. This is the generalization of the paper [W2] where the corresponding elliptic problem was studied.

The equations that we consider are the following

$$V_t - F(\mathrm{II}, v) = 0\,, \tag{1.1}$$

where $V_t$ is the normal velocity, II is the second fundamental form of a surface $S$ and $v$ is its normal. We assume that $F$ is uniformly elliptic along the solution surface and Lipschitz in $v$ and linear in II. Precise definitions of these terms are in Section 2.

The first example of this kind of equations is

$$V_t - F(\mathrm{II}, v) = V_t - tr\mathrm{II}\,, \tag{1.2}$$

whereas the corresponding solutions are the surfaces moving with their mean curvature.

The main result in this paper is to show that $S$ is regular if it is flat enough and has density close to 1. One of the main difficulties is that the area of $S$ does not make sense.

*2000 Mathematics Subject Classification*: 35K55, 53A10.
*Keywords*: Mean Curvature Flow, Regularity Theory, Parabolic Equation.