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ON THE SETS WHERE A CONTINUOUS FUNCTION HAS INFINITE ONE-SIDED DERIVATIVES

Abstract

In the present paper I give a characterization by help of measure and Borel classes of the set of points at which the continuous function possesses an infinite one-sided derivative. The main theorem is as follows. Let E_1 and E_2 be disjoint subsets of $\mathbb{R}$. There exists a continuous function $f : \mathbb{R} \to \mathbb{R}$ such that $E_1 = \{x : f'_+(x) = +\infty\}$ and $E_2 = \{x : f'_+(x) = -\infty\}$ if and only if (i) E_1 and E_2 are of type $F_{\alpha\delta}$ and measure zero and (ii) there exist disjoint sets F_1 and F_2 of type F_σ such that $E_1 \subset F_1$ and $E_2 \subset F_2$.

1 Introduction

In [4] and [2], the following theorems are proved:

Theorem I. *(Theorem 2 of [4]). Let E_1 and E_2 be disjoint subsets of $\mathbb{R}$. There exists a function $f : \mathbb{R} \to \mathbb{R}$ such that*

$$E_1 = \{x : f'_-(x) = +\infty\} \quad \textit{and} \quad E_2 = \{x : f'_-(x) = -\infty\}$$

if and only if $m(E_1) = m(E_2) = 0$, where f'_- is the left-hand derivative and m denotes the Lebesgue measure.
Analogously for the right-hand derivative.

Theorem II. *(The main theorem of [2]). Let E_1 and E_2 be disjoint subsets of $\mathbb{R}$. There exists a function $f : \mathbb{R} \to \mathbb{R}$ such that*

$$E_1 = \{x : f'(x) = +\infty\} \quad \textit{and} \quad E_2 = \{x : f'(x) = -\infty\}$$

if and only if

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