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SOME COMMENTS ON THE MCSHANE AND HENSTOCK INTEGRALS

Abstract

The basic distinction between the Henstock and McShane integrals involves the location of the tags. Another way to look at this difference is through the type of set associated with each tag; intervals for the Henstock integral and finite unions of intervals for the McShane integral. Does this distinction make any difference in the definition of the Riemann integral? What happens if the intervals are replaced with arbitrary measurable sets? These questions are answered in this paper. A relationship between the Henstock integral and outer Lebesgue measure is also included.

The Henstock integral and the McShane integral have similar definitions that are based upon allowing the mesh size in the Riemann integral definition to vary from point to point rather than to be constant. This rather subtle change from the Riemann definition has a great impact on what functions are integrable. The difference between the Henstock and McShane definitions involves the location of the tags. For the Henstock integral, the tag of an interval must belong to that interval whereas for the McShane integral the tag of an interval need not belong to the interval. Consequently, it is more difficult for a function to be McShane integrable and, as it turns out, the McShane definition kicks out those functions that are not absolutely integrable. It can be shown that the McShane integral is equivalent to the Lebesgue integral and that the Henstock integral is equivalent to the restricted Denjoy integral. The fact that such powerful integrals can be defined almost as simply as the Riemann integral is of pedagogical interest to some. The main focus in this paper will be the distinction between the two definitions.

The first question that will be examined is whether or not tags in versus tags out makes any difference when using a constant mesh size. Since the

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