Real Analysis Exchange
Vol. 23(1), 1998-99, pp. 325–327

Michal Málek*, Institute of Mathematics, Silesian University, 746 01 Opava, Czech Republic, e-mail:`Michal.Malek@fpf.slu.cz`

A COUNTEREXAMPLE OF AN EXTREMELY CHAOTIC FUNCTION

Abstract

We exhibit an example of continuous function f on the interval, which is extremely chaotic but not transitive, disproving a conjecture by Bruckner and Hu.

Bruckner and Hu stated the following result: *Under the Continuum Hypothesis, a continuous function f defined on $I_0 = [0,1]$ is chaotic if and only if f^2 is nomadic* [1].

Here "nomadic" means transitive and "chaos" means extremal chaos in the following sense. There is an uncountable set S such that, for every two different points $x, y \in S$,

$$\limsup_{n\to\infty} |f^n(x) - f^n(y)| = 1, \tag{1}$$

$$\liminf_{n\to\infty} |f^n(x) - f^n(y)| = 0. \tag{2}$$

In this note, we show that the assertion above is not true. Recall, that a function f on I_0 is called d-chaotic, if there exists an uncountable set S such that

$$\limsup_{n\to\infty} |f^n(x) - f^n(y)| = d \tag{3}$$

and (2) are satisfied for any two different points x, y in S. A basic set for f is any maximal infinite ω-limit set $\tilde{\omega}$ of f containing a periodic point. Such a set is indecomposible if, for every $n \in \mathbb{N}, \tilde{\omega}$ is a basic set of f^n.

Lemma 1. *A basic set $\tilde{\omega}$ is indecomposible if and only if for any interval J such that $J \cap \tilde{\omega}$ is infinite there exists $W = lim_{n\to\infty} f^n(J)$ with $\overline{W} \supset \tilde{\omega}$.*

Key Words: basic sets, extremal chaos, transitivity
Mathematical Reviews subject classification: 26A18, 58F13
Received by the editors July 22, 1997
*The research was supported, in part, by the Grant Agency of Czech Republic, grant No. 201/97/0001.