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ON COUNTABLY GENERATED INVARIANT σ-ALGEBRAS WHICH DO NOT ADMIT MEASURE TYPE FUNCTIONALS

Abstract

Using the Banach-Kuratowski matrix, we give a generalization of the result from [6] concerning the nonexistence of nonzero σ-finite diffused measures on some countably generated invariant σ-algebras of sets.

The present note is devoted to certain countably generated invariant σ-algebras of sets, which do not admit nonzero σ-finite diffused (i.e. continuous) measures. The main tool of this note is the so-called Banach-Kuratowski matrix (see [1]) which shows, in particular, that, under the Continuum Hypothesis, there exists a countably generated σ-algebra S of subsets of the real line $\mathbf{R}$, such that all one-element subsets of $\mathbf{R}$ belong to S and there is no nonzero σ-finite diffused measure defined on S.

Here we construct (under the Continuum Hypothesis, of course) a certain Banach-Kuratowski matrix consisting of almost invariant sets with respect to the group of all isometric transformations of $\mathbf{R}$. We then apply that matrix and obtain a generalization of a result given in the paper of Pelc and Prikry[6]. Notice that the method used in our further considerations is taken from the papers [4] and [5].

First of all let us recall the construction of a Banach-Kuratowski matrix. Let ω denote, as usual, the least infinite ordinal number and let $F = \omega^\omega$ denote the family of all functions acting from ω into ω. Let f and g be any two functions from F. We put $f \prec g$ if and only if there exists a natural

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