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AN ω-LIMIT SET FOR A LIPSCHITZ FUNCTION WITH ZERO TOPOLOGICAL ENTROPY

Abstract

Let Q be the middle thirds Cantor set in $[0,1]$, and take C to be the countable set containing the midpoints of the intervals complementary to Q, together with $\{-\frac{1}{6}\}$. We develop a Lipschitz function $f : [-\frac{1}{4},1] \to [-\frac{1}{4},1]$ that possesses zero topological entropy, and for which $Q \cup C$ – an uncountable set with isolated points – is an ω-limit set of f.

1 Introduction

The iterative properties of continuous functions have received considerable attention in recent years. In particular, much has been learned about the structure of the ω-limit sets that various classes of continuous functions possess. Bruckner and Smítal have characterized the structure of ω-limit sets for the class of continuous functions as well as those continuous functions with zero topological entropy [2], [3].

Theorem 1. *Let F be a nonempty closed set. Then F is an ω-limit set for a continuous function if and only if F is either nowhere dense, or F is a union of finitely many nondegenerate closed intervals.*

Theorem 2. *Let $F \subset (0,1)$ be a nonempty infinite closed set. Then F is an ω-limit set for a continuous function $f : [0,1] \to [0,1]$ with zero topological entropy if and only if $F = Q \cup C$, where Q is a Cantor set, and C is countable, dense in F if nonempty, and such that for any interval J contiguous to Q,* $\operatorname{card}(J \cap C) \le 1$ *if* 0 *or* 1 *is in* J, *and* $\operatorname{card}(J \cap C) \le 2$ *otherwise.*

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