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OMEGA–LIMIT SETS AND NON–CONTINUOUS FUNCTIONS

Abstract

We investigate the dense mapping property introduced by Keller in connection with iteration in Newton's method. Various kinds of functions are shown to have the dense mapping property. We show that a function has the dense mapping property iff it is bilaterally quasicontinuous. We also present an invariance theorem and other results on omega–limit sets.

Introduction and preliminaries

For a general background and notation, we refer the reader to [1], [4], [8]. We will let $f : \mathbb{R} \to \mathbb{R}$ represent a real-valued function on the real line. The iterates of f are defined inductively such that $f(f^n(x)) = f^{n+1}(x)$, where f^n is the n-fold composition of f. The *trajectory* of x in X is the sequence $\{f^n(x)\}_{n=0}^{\infty}$, where $f^0(x) = x$. The *orbit* of x is the point set $\{f^n(x) : n \geq 0\}$. The *omega–limit set* of f at x (denoted by $\omega(x, f)$) is the limit set of the sequence $\{f^n(x)\}_{n=0}^{\infty}$. Therefore

$$\omega(x, f) = \bigcap_{m \geq 0} \mathrm{Cl}\big(\cup_{n \geq m} f^n(x)\big)$$

where Cl( ) denotes the closure operator.

We investigate ω-limit sets of Darboux-Baire 1 functions and related functions, because of the familiar application to Newton's method of finding the zeros of a function which is differentiable, but not C^1.

A function $f : X \to Y$, where X and Y are topological spaces, is *quasicontinuous* at a point x in X if for any open set V containing $f(x)$, and for any open set U containing x, there exists an open nonempty set $G \subset U$ such

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