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LIGHTLY MIXING ON DENSE ALGEBRAS

Abstract

We use a set in the unit interval to construct measure preserving transformations that are lightly mixing on intervals but not ergodic, and ergodic measure preserving transformations that are lightly mixing on intervals but not lightly mixing, and give some applications.

1 Introduction

Let X denote the unit interval $[0, 1]$ and μ Lebesgue measure. A *transformation* is a map $T : X \to X$ that is one-to-one and onto a.e. such that T and T^{-1} are measurable. T is *measure preserving* if for all measurable sets A, $\mu(TA) = \mu(A)$. T is *ergodic* if for any measurable set A, $T(A) = A$ implies $\mu(A) = 0$ or 1. We identify sets that differ by a null set and write $A = B$ to mean $\mu(A \triangle B) = 0$.

Let $s = \{n_k\}$ be an infinite sequence. A finite measure preserving transformation T *sweeps out on s* if for all sets A of positive measure, $\bigcup_{k=0}^{\infty} T^{n_k} A = X$. Since X has finite measure, ergodicity is equivalent to the property that for all sets A of positive measure, T sweeps out on cofinite sequences. T is *lightly mixing* if for all sets A and B of positive measure

$$\liminf_{n\to\infty} \mu(T^n A \cap B) > 0.$$

It was shown in [2] that T sweeps out on all sequences if and only if T is lightly mixing. King [7] showed that the Cartesian product of lightly mixing transformations is lightly mixing, answering a question of Friedman; the question arises from [7] of whether there is a weak mixing transformation T such that T satisfies the lightly mixing condition on a dense algebra but is

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