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Roy A. Mimna, 57 West Liberty Street, Hubbard, Ohio 44425,
e-mail:mimna@aol.com

Eric J. Wingler, Department of Mathematics and Statistics, Youngstown State University, Youngstown, Ohio 44555, e-mail:wingler@math.ysu.edu

LOCALLY BOUNDED FUNCTIONS

1 Introduction

A well known property of continuous real-valued functions $f : X \to \mathbf{R}$, where X is a topological space, is that for any $x \in X$, f is bounded on some neighborhood of x. Here we discuss this property of continuous functions, which we shall call "local boundedness", in a more general context. R. V. Fuller's "subcontinuity" is shown to be equivalent to local boundedness. The classical theorem that a continuous real-valued function on a compact space is bounded, is generalized and shown to be true for the larger class of locally bounded functions. Certain classes of discontinuous functions are shown to be locally bounded, and properties of locally bounded functions are studied.

Throughout this paper, X and Y denote topological spaces, with no special properties unless otherwise indicated.

2 Basic Definitions

Definition 1. *A function $f : X \to M$, where M is a metric space, is locally bounded if for any x in X there exists an open set U containing x such that f is bounded on U.*

If $f : X \to M$ is a continuous function and x is any point in X, then f is bounded on the open set $f^{-1}[B(f(x), 1)]$, where $B(f(x), 1)$ denotes an open ball in M of radius 1. Hence we have the following theorem.

Theorem 1. *If M is a metric space and $f : X \to M$ is a continuous function, then f is locally bounded.*

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