Real Analysis Exchange
Vol. 23(1), 1998-99, pp. 247–251

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ON THE CONVERGENCE OF THE INTEGRALS OF A TRUNCATED HENSTOCK-KURZWEIL INTEGRABLE FUNCTION

Abstract

We deal with two ways to truncate a Henstock-Kurzweil integrable function and the convergence of their integrals. We also give an example to show the limitations to the convergence theorem.

The problem we deal with here is the following. Let $f : [a, b] \longrightarrow \mathbb{R}$ be a measurable function. For M and N positive real numbers, we define two *truncations* of f:

$$f_{M,N}(x) = \begin{cases} f(x) & \text{if } -N \le f(x) \le M \\ M & \text{if } f(x) \ge M \\ -N & \text{if } -N \ge f(x) \end{cases}$$

$$\tilde{f}_{M,N}(x) = \begin{cases} f(x) & \text{if } -N \le f(x) \le M \\ 0 & \text{if } f(x) \ge M \\ 0 & \text{if } -N \ge f(x) \end{cases}$$

If M, N go to infinity both $f_{M,N}$ and $\tilde{f}_{M,N}$ converge pointwise to f. So if f is Lebesgue integrable, by the Dominated Convergence Theorem, their integrals converge to that of f. In particular if we take $M = N$. But this does not happen in the case of the Henstock-Kurzweil integral. In [2, Section 18, pp. 114–118] there is a study of the cases when $\int \tilde{f}_{M,M}$ converges to $\int f$. The

Key Words: convergence, Henstock-Kurzweil integral, truncation
Mathematical Reviews subject classification: 26A39
Received by the editors September 16, 1996