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THE PREVALENT DIMENSION OF GRAPHS

Abstract

We show that the set of functions in $C[0,1]$ with a graph of packing dimension 2 (or, equivalently, upper entropy dimension 2) is prevalent.

1 Prevalence

In his excellent monograph *Measure and Category* [8], Oxtoby compares and contrasts the most familiar two notions of "almost nowhere" on the real line. The extension of these ideas, Lebesgue measure zero and Baire first category, to infinite dimensional spaces is an interesting problem.

The notions of Baire category extend immediately to any complete, separable metric space and, in particular, to $C[0,1]$. A set is said to be of *first category* or *meager* if it may be expressed as a countable union of nowhere dense sets. A set is said to be *generic* or *comeager* if it is the complement of a meager set. A classic theorem of Banach states that the set of functions in $C[0,1]$ which are nowhere differentiable forms a comeager subset ([8] chapter 11). This is frequently phrased as, the generic continuous function is nowhere differentiable. As another example, Humke and Petruska [4] prove that the set of functions in $C[0,1]$ whose graph has lower entropy index one is comeager and the set of functions in $C[0,1]$ whose graph has upper entropy dimension two is comeager. See section 2 for definitions. Their statement that the generic function in $C[0,1]$ has a graph with lower entropy index 1 strengthens a theorem of Mauldin and Williams which states that the generic function in $C[0,1]$ has a graph with Hausdorff dimension 1 ([7] Theorem 2).

There are fundamental difficulties, however, with attempts to extend measures to infinite dimensional spaces. Prevalence is a notion defined in [5] which generalizes the measure theoretic "almost nowhere" without actually defining a measure on the entire space. An equivalent notion was originally introduced

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