INROADS

Real Analysis Exchange
Vol. 23(1), 1998-99, pp. 235–240

Hanxiang Chen, Department of Mathematics, University of Alabama
1310 37 th St. E. Apt. 92, Tuscaloosa, AL 35405,
e-mail:hchen3@ua1vm.ua.edu

A PAIR OF ADJOINT CLASSES OF RIEMANN–STIELTJES INTEGRABLE FUNCTIONS

Abstract

The purpose of this paper is to show that the classes of Riemann integrable functions and absolutely continuous functions are adjoint with respect to the $(R\text{-}S)$ integral $\int_a^b f\,dg$.

Definition. Let A and B be two classes of functions defined on $[a,b]$. A and B are said to be adjoint with respect to the $(R\text{-}S)\int_a^b f\,dg$, if the following conditions are satisfied:

(i) If $f \in A$ and $g \in B$, then the $(R\text{-}S)\int_a^b f\,dg$ exists;

(ii) If the $(R\text{-}S)\int_a^b f\,dg$ exists for all $g \in B$, then $f \in A$; and

(iii) If the $(R\text{-}S)\int_a^b f\,dg$ exists for all $f \in A$, then $g \in B$.

If A and B are adjoint with respect to the $(R\text{-}S)\int_a^b f\,dg$, this means that on condition that the $(R\text{-}S)\int_a^b f\,dg$ exists, neither A nor B can be extended at all. For convenience, we write $(A * B)\int_a^b f\,dg$ meaning that A and B are adjoint with respect to the $(R\text{-}S)\int_a^b f\,dg$.

We introduce the following symbols for some classes of functions defined on $[a,b]$:

Key Words: adjoint classes, Riemann integrable functions, the Riemann-Stieltjes integral, functions of bounded variation, absolutely continuous functions
Mathematical Reviews subject classification: 26A42
Received by the editors May 26, 1996