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ON THE NON-EXISTENCE OF CERTAIN BOUNDED LINEAR PROJECTIONS

Abstract

It is known that there is a bounded linear operator A from the space of bounded real functions to the subspace of bounded Lebesgue-measurable functions such that for any Lebesgue-measurable function f we have $Af = f$ for a.e. $x \in \mathbb{R}$. S. A. Argyros proved that A could not be a projection; i.e. we can always find a bounded measurable function f and a point $x \in \mathbb{R}$ for which $(Af)(x) \neq f(x)$.

We give an independent proof and in particular we prove that there does not exist a projection to the space of functions with the Baire property, either.

S. A. Argyros proved in [AR] that there does not exist a bounded linear projection from the space of all bounded real functions to the subspace of all bounded Lebesgue-measurable functions and the subspace of all bounded Borel-measurable functions.

In this paper we give an independent proof, and our proof covers more general cases as well. In particular, we prove that such a projection does not exist to the subspace of functions with the Baire property, either.

More precisely, we show that if $\mathcal{M} \subseteq P(\mathbb{R})$ is a σ-algebra, if there is a σ-ideal $\mathcal{K} \subset \mathcal{M}$, $\mathcal{K} \neq \mathcal{M}$ and if $\mathcal{P} \subseteq \mathcal{N} \stackrel{\text{def}}{=} \mathcal{M} \setminus \mathcal{K}$ such that

(0) $\{x\} \in \mathcal{M}$ for all $x \in \mathbb{R}$;

(1) for every $N \in \mathcal{N}$ there exists $P \subseteq N, P \in \mathcal{P}$;

(2) given more than ω elements of $\mathcal{N}$ there exist infinitely many of them with non-empty intersection;

(3) $|\mathcal{P}| \leq 2^\omega$ and $|P| = 2^\omega$ for all $P \in \mathcal{P}$,

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