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COMPOSITIONS OF DARBOUX-LIKE FUNCTIONS

Abstract

An example is given of a real connectivity function which is not the composition of any finite collection of almost continuous functions. We also investigate conditions under which the composition of two real Darboux functions can be continuous.

In [6] I asked if every Darboux function from $\mathbb{R}$ to $\mathbb{R}$ is the composition of two almost continuous functions. In the present note it is shown that the answer is "no". In fact, there exits a connectivity function which cannot be written as the composition of finitely many almost continuous functions. This example is not particularly difficult. I think we have overlooked this example until now because no one expected it to exist. The composition of almost continuous functions can be very nasty. Our function which is not such a composition is as well behaved as a Darboux function can be and not be almost continuous.

Natkaniec [7] has shown that if f is Darboux and nasty–that is, $f^{-1}(x)$ is c-dense for every x, then f is the composition of two almost continuous functions. To look at the other extreme, we need a Darboux function which is not almost continuous but as nice as possible. Since a Darboux function of Baire class 1 is almost continuous [1], the function we want must be totally discontinuous on some perfect set. Jones and Thomas [5] give an example of a function which is connectivity, continuous on the complement of the Cantor set, but not almost continuous. Our example is a simple modification of the Jones and Thomas example.

A function $f : \mathbb{R} \to \mathbb{R}$ is *almost continuous* if, given a closed set $K \subset \mathbb{R}^2$ such that $\text{graph}(f) \cap K = \emptyset$, there exists a continuous function $g : \mathbb{R} \to \mathbb{R}$ such that $\text{graph}(g) \cap K = \emptyset$. f is *Darboux* if $f(C)$ is connected whenever C is connected. f is a *connectivity function* if $\text{graph}(f|_C)$ is connected whenever C

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