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ON THE MEASURABILITY OF FUNCTIONS DEFINED ON THE PRODUCT OF TWO TOPOLOGICAL SPACES

Abstract

Some conditions implying the measurability of functions defined on the product of two topological spaces are investigated.

Let $\mathbb{R}$ denote the set of all reals and let $(X, \mathcal{T}_X)$ and $(Y, \mathcal{T}_Y)$ be topological spaces. Moreover, let μ_1 and μ_2 respectively, be σ-finite measures defined on some σ-fields $\mathcal{M}_1 \supset \mathcal{T}_X$ and $\mathcal{M}_2 \supset \mathcal{T}_Y$. Assume that

(1) for every set $A \in \mathcal{M}_1$ with $\mu_1(A) > 0$ there is a set $B \in \mathcal{T}_X$ such that $B \subset A$ and $\mu_1(B) > 0$;

(2) $\mu_1(A) > 0$ for all nonempty sets $A \in \mathcal{T}_X$.

A function $f : X \to \mathbb{R}$ is called $\mathcal{T}_X$-quasicontinuous ($\mathcal{T}_X$-cliquish) at a point $x \in X$ ([5] if for every positive real η and for every set $U \in \mathcal{T}_X$ containing x there is a nonempty set $V \in \mathcal{T}_X$ such that $V \subset U$ and $|f(v) - f(x)| < \eta$ for all points $v \in V$ ($\operatorname{osc}_V f < \eta$, where $\operatorname{osc}_V f$ denotes the diameter of the set $f(V)$).

In the proofs we will use the following Davies lemma ([2, 3]):

Lemma 1. *Suppose that the measure μ_1 is complete and a function $f : X \to \mathbb{R}$ is such that for every positive real η and for every set $A \in \mathcal{M}_1$ with $\mu_1(A) > 0$ there is a set $B \in \mathcal{M}_1$ such that $B \subset A$, $\mu_1(B) > 0$ and $\operatorname{osc}_B f < \eta$. Then the function f is μ_1-measurable.*

Remark 2. *If a function $f : X \to \mathbb{R}$ is measurable with respect to μ_1, then it is $\mathcal{T}_X$-cliquish at every point $x \in X$;*

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