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Dariusz Banaszewski,* Mathematics Department, Pedagogical University, Chodkiewicza 30, 85–064 Bydgoszcz, Poland.
Krzysztof Ciesielski,† Department of Mathematics, West Virginia University, Morgantown, WV 26506–6310, e-mail: kcies@wvnvms.wvnet.edu.XS

COMPOSITIONS OF TWO ADDITIVE ALMOST CONTINUOUS FUNCTIONS

Abstract

In the paper we prove that an additive Darboux function $f\colon \mathbb{R} \to \mathbb{R}$ can be expressed as a composition of two additive almost continuous (connectivity) functions if and only if either f is almost continuous (connectivity) function or $\dim(\ker(f)) \neq 1$. We also show that for every cardinal number $\lambda \leq 2^\omega$ there exists an additive almost continuous functions with $\dim(\ker(f)) = \lambda$. A question whether every Darboux function $f\colon \mathbb{R} \to \mathbb{R}$ can be expressed as a composition of two almost continuous functions (see [**?**] or [**?**]) remains open.

1 Definitions and Notation

Our terminology and notation is standard. In particular, functions will be identified with their graphs, and for a subset A of $\mathbb{R} \times \mathbb{R}$ (possibly, but not necessarily, a graph of a function) we will write $\mathrm{dom}\,(A)$ and $\mathrm{rng}\,(A)$ to denote the x-projection (the domain) and the y-projection (the range) of A, respectively. The cardinality of a set A will be denoted by $\mathrm{card}\,(A)$. Cardinals will be identified with the initial ordinals. The cardinality of the set $\mathbb{R}$ of real numbers, the continuum, will be denoted by 2^ω.

Throughout the paper we will consider $\mathbb{R}$ as a linear space over the field $\mathbb{Q}$ of rational numbers. A linear basis of this space will be referred to as a *Hamel basis*. It is evident that the cardinality of every Hamel basis is equal to 2^ω.

Key Words: Darboux function, connectivity function, almost continuous function, additive function, composition of functions.
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