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ON UNBOUNDED ORDER CONVERGENCE

Abstract

Order convergence in Riesz spaces is defined only for order bounded sets; so does not apply, for example, to pointwise convergence in function spaces. There exists a generalization to not necessarily order bounced nets that includes the above pointwise convergence. In practice, application of this generalization is rather involved. We show that in the case of a Riesz space with a weak order unit, the definition takes a quite simple form, readily applicable to proofs.

A central tool in the study of Riesz spaces is order convergence. This is defined only for order bounded sets, so cannot be applied to situations in which the nets concerned are not necessarily order bounded. For example, it does not include pointwise convergence of nets in a Riesz space of functions on a set X.

There exists a generalization of order convergence which applies to all nets in a Riesz space and includes, among others, the above pointwise convergence (and reduces, of course, to order convergence for order bounded nets). To my knowledge, it was first defined and applied by H. Nakano ([4], [5]) (under the name "individual convergence" in the second reference). Following DeMarr [1], we will call it "unbounded order convergence".

The various definitions of unbounded order convergence do not lend themselves easily to proofs of theorems. However, in our work on the Lebesgue integral in the bidual of $C(X)$ for X compact (cf. [3]), and in a planned sequel on measurability in a superspace of the bidual, we deal with spaces – Dedekind complete spaces with a weak order unit – in which unbounded order convergence has a simple characterization. We present the details in the present note.

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