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CONVERGENCE AND APPROXIMATE DIFFERENTIATION

Abstract

The main result of this paper is Theorem 1, which states the following: Let $F, F_n : [a,b] \to \mathbb{R}$, $n = 1, 2, \ldots$ be Lebesgue measurable functions such that $\{F_n\}_n$ converges pointwise to F on $[a,b]$. If each F_n is approximately derivable *a.e.* on $[a,b]$, $\{F_n\}_n$ is uniformly absolutely continuous on a set $P \subset [a,b]$, and $\{(F_n)'_{ap}\}_n$ converges in measure to a measurable function g, finite *a.e.* on $[a,b]$, then F is approximately derivable *a.e.* on P and $F'_{ap}(x) = g(x)$ *a.e.* on P. An immediate consequence of this result is the famous theorem of Džvaršeĭšvili on the passage to the limit for the Denjoy and Denjoy* integrals (see Theorem 47, p. 40 of [3]). As was pointed out by Bullen in [3] (p. 309), "the $\mathcal{D}^*$ integral case of Theorem 47 of [3] was rediscovered by Lee P. Y." (see also Theorem 7.6 of [7]).

1 Preliminaries

We shall denote the Lebesgue measure of the set A by $m(A)$, whenever $A \subset \mathbb{R}$ is Lebesgue measurable. If $f : [a,b] \to \mathbb{R}$ and $[\alpha, \beta] \subseteq [a,b]$, then let $\mathcal{O}(F; [\alpha, \beta] = \sup\{|F(y) - F(x)| \ : \ x, y \in [\alpha, \beta]\}$. Let $\mathcal{C}$ denote the class of all continuous functions and $\mathcal{C}_{ap}$ the class of all approximately continuous functions. A function $f : P \to \mathbb{R}$ is said to satisfy Lusin's condition (N), if $m(F(Z)) = 0$, whenever $m(Z) = 0$. For the definitions of AC, AC^*, VB and VB^* see [11].

Definition 1. *([11], p. 221).* Let $F : P \to \mathbb{R}$, and $Q \subseteq P$. We denote by $V(F; Q)$ the upper bound of the numbers $\sum_i |F(b_i) - F(a_i)|$, where $\{[a_i, b_i]\}_i$ is any sequence of nonoverlapping closed intervals with endpoints in Q. (We may suppose without loss of generality that $\{[a_i, b_i]\}_i$ is a finite set.)

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