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DESCRIPTIVE CHARACTER OF SETS OF DENSITY AND $\mathcal{I}$-DENSITY POINTS

Abstract

Let $X = [a, b]$ and $A \subset X^2$. We extend the theorem of Mauldin stating the set of $\langle x, y\rangle \in X^2$ such that y is a density point of A_x, provided that A is Borel is itself a Borel set. We prove the corresponding result if A is analytic or coanalytic and show the analogous statements in the category case.

1 Introduction

Let $X = [a, b]$. If $E \subset X$ is a Lebesgue measurable set, $\varphi(E)$ denotes the set of all density points of E. If $E \subset X$ possesses the Baire property, $\varphi_{\mathcal{I}}(E)$ denotes the set of all $\mathcal{I}$-density points, i.e., the density points in the sense of category, introduced by Wilczyński in [W]. For $A \subset X^2$ and $x \in X$, we put

$$A_x = \{y \in X : \langle x, y\rangle \in A\};$$

the so-called x-section of A. By LM_k (respectively, BP_k) we denote the class of Lebesgue measurable sets (sets with the Baire property) in $\mathbb{R}^k$ for $k = 1, 2$. For $A \subset X^2$ we put

$$D(A) = \{\langle x, y\rangle \in X^2 : A_x \in \mathrm{LM}_1 \ \&\ y \in \varphi(A_x)\};$$

$$D_{\mathcal{I}}(A) = \{\langle x, y\rangle \in X^2 : A_x \in \mathrm{BP}_1 \ \&\ y \in \varphi_{\mathcal{I}}(A_x)\}.$$

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