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## ON PRODUCTS OF QUASICONTINUOUS FUNCTIONS

### Abstract

It is shown that every cliquish function  $f$  defined on a pseudometrizable space for which the preimages of the positive and the negative half-axes are simply open sets is the product of two quasicontinuous functions. If moreover  $f$  is in the Baire class  $\alpha$ , then we can take the factors also in the Baire class  $\alpha$ .

### 1 Introduction

In 1985 in [5] Z. Grande showed that there is a cliquish function of a real variable which cannot be written as the finite product of quasicontinuous functions and asked for characterization of such functions. This characterization has been given by T. Natkaniec in 1990 in [10], where it is shown that a function  $f: \mathbb{R} \rightarrow \mathbb{R}$  can be factored into a finite product of quasicontinuous functions if and only if it is cliquish and

( $\star$ ) each of the sets  $f^{-1}((-\infty, 0))$ ,  $f^{-1}((0, \infty))$  and  $f^{-1}(0)$  is simply open.

However, in his representations of such function he used as many as eight quasicontinuous functions. This result is generalized in [2] where it is shown that each real cliquish function defined on a separable metrizable space satisfying the condition ( $\star$ ) can be written as the product of three quasicontinuous functions. Moreover, it is shown that the condition ( $\star$ ) can be replaced with “the set  $f^{-1}(0)$  is simply open” for a Baire space. Finally, A. Maliszewski in [8] (see also [7]) proved that two quasicontinuous functions are sufficient for a cliquish function  $f$  defined on  $\mathbb{R}^n$  satisfying ( $\star$ ). In this paper we generalize this result for a cliquish function satisfying ( $\star$ ) defined on a pseudometrizable space. The proof is quite different from [8].

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