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## A DESCRIPTIVE DEFINITION OF THE KH-STIELTJES INTEGRAL<sup>†</sup>

### Abstract

This paper gives a descriptive definition of Stieltjes integrals (on a compact interval of the real line) in the frame of Kurzweil-Henstock integration. Five conditions characterize the functions that are an indefinite integral with respect to some continuous function of generalized bounded variation.

### 1 Introduction

A descriptive definition of the Kurzweil-Henstock integral, involving differentiability almost everywhere together with some null condition, is known since a few years (cf. for instance [3]). A more complete fundamental theorem was given by W. B. Jurkat and R. W. Knizia for the multidimensional weak integral in [4] and [5], where these authors introduced a useful and natural outer measure associated to any (interval) function.

In a preceding paper [1], I gave such a fundamental theorem for the multidimensional integrals of J. Mawhin [6] and W. F. Pfeffer [8]. In the present one, I propose a similar theorem for the Kurzweil-Henstock-Stieltjes integral on a compact interval  $[a, b] \subseteq \mathbb{R}$ . Five equivalent conditions thus characterize the functions  $F : [a, b] \rightarrow \mathbb{R}$  which are an indefinite integral of some function  $f : [a, b] \rightarrow \mathbb{R}$  relatively to  $U : [a, b] \rightarrow \mathbb{R}$ , cf. Theorem 4.7 and Corollary 5.6. The function  $U$  is assumed to be continuous and VBG<sup>o</sup> (equivalently, VBG\* in the sense of Saks [9]).

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