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A STABILITY RESULT FOR NONLINEAR NEUMANN PROBLEMS IN REIFENBERG FLAT DOMAINS IN $\mathbb{R}^N$

Antoine Lemenant and Emmanouil Milakis

Abstract

In this paper we prove that if Ω_k is a sequence of Reifenberg-flat domains in $\mathbb{R}^N$ that converges to Ω for the complementary Hausdorff distance and if in addition the sequence Ω_k has a "uniform size of holes", then the solutions u_k of a Neumann problem of the form

$$\begin{cases} -\operatorname{div} a(x,\nabla u_k) + b(x,u_k) = 0 & \text{in } \Omega_k \\ a(x,\nabla u_k)\cdot \nu = 0 & \text{on } \partial\Omega_k \end{cases} \tag{0.1}$$

converge to the solution u of the same Neumann problem in Ω. The result is obtained by proving the Mosco convergence of some Sobolev spaces, that follows from the extension property of Reifenberg-flat domains.

Introduction

In this paper we study the stability of solutions for the following nonlinear Neumann problem

$$\begin{cases} -\operatorname{div} a(x,\nabla u) + b(x,u) = 0 & \text{in } \Omega \\ a(x,\nabla u)\cdot \nu = 0 & \text{on } \partial\Omega \end{cases} \tag{0.2}$$

where Ω is a bounded subset of $\mathbb{R}^N$, $a\colon \mathbb{R}^N\times\mathbb{R}^N \to \mathbb{R}^N$ and $b\colon \mathbb{R}^N\times\mathbb{R} \to \mathbb{R}$ are two Carathéodory functions satisfying suitable monotonicity, coerciveness and growth conditions (see (1.1)–(1.3) below). More precisely, we are interested in the following question. Let Ω_k be a sequence of open sets in $\mathbb{R}^N$ that converges to Ω for the complementary Hausdorff distance. Let u_k be the sequence of solutions for the problem (0.2) in Ω_k and let u be the solution associated to Ω. Is it true that u_k converges to u? If the answer is positive we say that the problem (0.2) is stable along the sequence Ω_k (see Definition 3).

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