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LOCAL AND NONLOCAL WEIGHTED p-LAPLACIAN EVOLUTION EQUATIONS WITH NEUMANN BOUNDARY CONDITIONS

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Abstract

In this paper we study existence and uniqueness of solutions to the local diffusion equation with Neumann boundary conditions and a bounded nonhomogeneous diffusion coefficient $g \geq 0$,

$$\begin{cases} u_t = \operatorname{div}\left(g|\nabla u|^{p-2}\nabla u\right) & \text{in }]0,T[\times\Omega, \\ g|\nabla u|^{p-2}\nabla u \cdot \eta = 0 & \text{on }]0,T[\times\partial\Omega, \end{cases}$$

for $1 \leq p < \infty$. We show that a nonlocal counterpart of this diffusion problem is

$$u_t(t,x) = \int_\Omega J(x-y) g\left(\frac{x+y}{2}\right) |u(t,y)-u(t,x)|^{p-2}(u(t,y)-u(t,x))\, dy \quad \text{in }]0,T[\times\Omega,$$

where the diffusion coefficient has been reinterpreted by means of the values of g at the point $\frac{x+y}{2}$ in the integral operator. The fact that $g \geq 0$ is allowed to vanish in a set of positive measure involves subtle difficulties, specially in the case $p = 1$.

1. Introduction

We consider the p-Laplacian evolution equation with homogeneous Neumann boundary conditions and a bounded nonhomogeneous diffusion coefficient $g \geq 0$, that is

$$N_p^g(u_0) \begin{cases} u_t = \operatorname{div}\left(g|\nabla u|^{p-2}\nabla u\right) & \text{in }]0,T[\times\Omega, \\ g|\nabla u|^{p-2}\nabla u \cdot \eta = 0 & \text{on }]0,T[\times\partial\Omega, \\ u(x,0) = u_0(x) & \text{in } \Omega, \end{cases}$$

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Key words. Nonlocal diffusion, p-Laplacian, total variation flow, Neumann boundary conditions.