THE SLOW STEADY MOTION OF LIQUID PAST A SEMI-ELLIPTICAL BOSS

G. POWER AND D. L. SCOTT-HUTTON

1. Introduction. In this problem of two-dimensional viscous flow, liquid is supposed to have a rigid boundary represented by ABCDE in Figure 1 and, apart from the disturbance caused by the presence of the elliptical boss BCD, is assumed to be in uniform shearing motion. The stream function is thus a biharmonic function vanishing together with its normal derivative at all points of the boundary, and proportional to y^2 at a great distance from the boss. A series of functions is found, each of which satisfies all the boundary conditions save one. A linear combination of these functions will also satisfy the boundary conditions with this one exception, and by a particular choice of the arbitrary constants which it contains, the remaining condition can be satisfied at as many points as desired. Special cases are discussed, and a process of approximation is outlined which yields the most accurate results at C, and also gives a convenient function for determining at any point of the boundary the magnitude of the error in the unsatisfied boundary condition. A special case of this problem has previously been considered [1].



Figure 1.

2. The stream function. We take the equation of the boundary BCD to be $x^2/a^2 + y^2/b^2 = 1$, and note that the region occupied by the fluid, for which y is never negative, is transformed into the interior of the semi-circle of unit radius shown in Figure 2 by

(1)
$$-2z = (a-b)w + (a+b)/w$$

The stream function ψ is biharmonic, that is to say it must satisfy $\nabla^{4}\psi=0$, and a satisfactory solution to the problem is

$$(2) \qquad \qquad \psi = y^2 + U + yV,$$

Received September 14, 1954.