THE STRICT DETERMINATENESS OF CERTAIN INFINITE GAMES

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1. Introduction. Gale and Stewart [1] have discussed an infinite two-person game in extensive form which is the generalization of a game as defined by Kuhn [3] obtained by deleting the requirement of finiteness of the game tree and regarding as plays all unicursal paths of maximal length originating in the distinguished vertex x_0. In a *win-lose* game the set S of all plays is divided into two sets S_I and S_{II} such that player I wins the play s if $s \in S_I$ and player II wins it if $s \in S_{II}$. Gale and Stewart have shown that a two-person infinite win-lose game of perfect information with no chance moves (called a GS game here) is strictly determined if S_I belongs to the smallest Boolean algebra containing the open sets of a certain topology for S. Here we answer affirmatively the question posed by them: Is a GS game strictly determined if S_I is a G_δ (or, equivalently, an F_σ)? The notation and results of [1] are used throughout, as well as the partial ordering of X given by: $x > y$ if $f^n(x) = y$ for some $n \geqslant 1$.

2. Alternative description of S_I. Let Γ be the game $(x_0, X_I, X_{II}$ $X, f, S, S_I, S_{II})$, where

$$S_I = \bigcap_{n=1}^{\infty} E_n ,$$

$E_1 \supseteq E_2 \supseteq \cdots$, and E_n is open. Following [3], let the rank $rk(x)$, for $x \in X$, be the unique k such that $f^k(x) = x_0$. As in [1], $\mathfrak{U}(x)$ is the set of all plays passing through x (the topology for S is that in which $\mathfrak{U}(x)$ is a neighborhood of each play in it). Then for each n,

$$E_n = \bigcup \{\mathfrak{U}(y) : \mathfrak{U}(y) \subseteq E_n\} ;$$

and since for any $y \in X$ we have

$$\mathfrak{U}(y) = \bigcup \{\mathfrak{U}(z) : f(z) = y\} ,$$

with

$$rk(z) = 1 + rk(y) ,$$

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