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SOME INCLUSIONS IN MULTIPLIERS

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G is a compact abelian group. The main result of this paper is that if T is a $(p, 1)$ multiplier, $1 < p \leqq 2$, then T is a (p, s) multiplier for all s in the range $1 \leqq s < p$ and also an (r, r) multiplier for $p < r < p'$ (p' conjugate of p).

An operator T defined on $L^p(G)$, whose range lies in the set of measurable functions on G is said to be of *weak type* (p, q) if there is a number A such that

$$m(\{x \in G: |Tf(x)| > t\}) \leqq \left(\frac{A\|f\|_p}{t}\right)^q$$

for all $f \in L^p$ and all $t > 0$. (m is Haar measure.) T need not be linear.

A linear operator, defined on $L^p(G)$ is said to be of *strong type* (p, q) if there exists a number A such that

$$\|Tf\|_q \leqq A\|f\|_p .$$

If T is of strong type (p, q) and commutes with translations (or equivalently with convolutions), then T is called a (p, q) *multiplier*. In this case we write $T \in M_p^q$. The Banach space of (p, p) multipliers is denoted M_p.

If $T \in M_p^q$, then there is a function φ on the dual of G such that $(Tf)^\wedge = \varphi\hat{f}$, for all $f \in L^p$, where $^\wedge$ denotes the Fourier transformation. T and φ are in one-to-one correspondence and we shall often write T_φ for T.

Using a deep theorem of E. M. Stein [3] on limits of sequences of operators we prove the following theorems:

THEOREM 1. *If $T \in M_p^1$, $1 \leqq p \leqq 2$ then T is of weak type (p, p).*

THEOREM 2. (*converse.*) *Let T be a linear map of L^p, $1 < p \leqq 2$ which commutes with translation and is of weak type (p, p). Then T is of strong type (p, s) for all s in the range $1 \leqq s < p$.*

These theorems imply the following corollaries.

COROLLARY 1. *If $T \in M_p^1$, $1 < p \leqq 2$, then $T \in M_p^s$ for all s in the range $1 \leqq s < p$.*

COROLLARY 2. *If $T \in M_p^1$, $1 \leqq p \leqq 2$, then $T \in M_r$ for all r in the*